

Simulation of Backstepping-based Nonlinear Control for Quadrotor Helicopter

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Abstract: The control of quadrotor helicopter in a hover condition has always been a great challenge for control engineers and researchers. Various algorithms have been developed to control this type of helicopter due to its high nonlinearities. This paper presents the dynamic model of quadrotor helicopter together with backstepping-based nonlinear control design which stabilizes the system. In order to verify the feasibility of the proposed controller, extensive simulations are performed using Matlab Simulink software. Here, the proper method to configure the solver options of the model design to provide an efficient and accurate simulation performance is presented. Through simulation test by using the dynamic model of a four degree of freedom quadrotor helicopter, the proposed approach can effectively perform stabilization and trajectory tracking. Besides, the simulation results indicate that the proposed design techniques can stabilize the quadrotor helicopter with better performance as compared to the traditional control schemes.

Keywords: Backstepping control; Matlab Simulink; Quadrotor helicopter; Simulation.

1. INTRODUCTION

Recent interest in the utilization of unmanned aerial vehicles (UAVs) in a variety of civil and military applications has prompted the need for such systems to operate with increased levels of autonomy. The UAVs have shown applications in different areas including search and rescue (SAR), meteorological studies, infrastructure inspection, homeland security and traffic surveillance.

Several methods have been proposed to control a quadrotor vehicle. In Hamel et al. [1] a simplified model of the X-4 Flyer dynamics is presented. Both Pound et al. [2] and McKerrow et al. [3] used this model to treat quadrotor control. Castillo et al. [4] applied a linear quadratic regulator (LQR) on a quadrotor platform. In Salih et al. [5] a proportional-integral-derivative (PID) controller is considered to stabilize the quadrotor helicopter. Sangyam et al. [6] employed a self-tuning PID control algorithm for quadrotor path tracking. There are other publications [7-10] which simplified the model by ignoring the gyroscopic effects and used a PID control technique. In these works, it can be found that many of the proposed control systems are based on a linearized model with conventional PID or linear control. Such techniques require the linearization of the system dynamics which will result in a loss of robustness of the control of the vehicle. Thus, nonlinear control scheme is introduced to handle nonlinear system more effectively and achieve good stability and tracking performance.

The backstepping control scheme is a nonlinear control method based on the Lyapunov theorem. The backstepping control design techniques have received great attention because of its systematic and recursive design methodology for nonlinear feedback control [11-12]. The backstepping approach offers a choice of design tools for accommodation of nonlinearities, and can avoid unwanted cancellations. The advantage of backstepping compared with other control methods lies in its design flexibility, due to its recursive use of Lyapunov functions. The key idea of the backstepping design is to select recursively some appropriate state variables as virtual inputs for lower dimension subsystems of the overall system and the Lyapunov functions are designed for each stable virtual controller [13]. Therefore, the final design of the actual control law can guarantee the stability of total control system.

The performance of the proposed controller is evaluated by performing a number of simulations using Matlab Simulink software. By using the standard configuration of Matlab Simulink, a quadrotor simulation platform is developed to assist in the design, development and validation of controllers. The dynamic model of a four degree of freedom (DOF) quadrotor helicopter is used to construct the simulation environment and then is employed to test the control design to stabilize the vehicle in a hovering mode. The latter provides a development of four DOF nonlinear quadrotor helicopter models.

2. CONTROL SYSTEM FOR QUADROTOR

The quadrotor helicopter considered in this study is illustrated in Figure 1. The dynamical equations describing the quadrotor rotorcraft are given in Equation (1), where the first three of this equation describe the dynamics in the Cartesian space, whereas the last three express the dynamics in the Euler angles, i.e., the attitude [14].

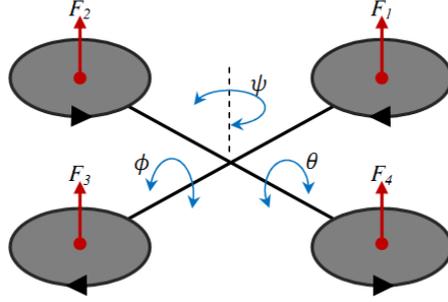


Figure 1. Quadrotor helicopter configuration

$$\begin{aligned}
\ddot{x} &= (c_\phi s_\theta c_\psi + s_\phi s_\psi) \frac{1}{m} u_1 \\
\ddot{y} &= (c_\phi s_\theta c_\psi - s_\phi c_\psi) \frac{1}{m} u_1 \\
\ddot{z} &= -g + (c_\phi c_\theta) \frac{1}{m} u_1 \\
\ddot{\phi} &= \dot{\theta} \dot{\psi} \left(\frac{I_{yy} - I_{zz}}{I_{xx}} \right) - \frac{J_r}{I_{xx}} \dot{\theta} \Omega_d + \frac{l}{I_{xx}} u_2 \\
\ddot{\theta} &= \dot{\phi} \dot{\psi} \left(\frac{I_{zz} - I_{xx}}{I_{yy}} \right) + \frac{J_r}{I_{yy}} \dot{\phi} \Omega_d + \frac{l}{I_{yy}} u_3 \\
\ddot{\psi} &= \dot{\theta} \dot{\phi} \left(\frac{I_{xx} - I_{yy}}{I_{zz}} \right) + \frac{1}{I_{zz}} u_4
\end{aligned} \tag{1}$$

In this paper, for hovering control, the system is simplified into four DOF i.e. only the z-directional linear motion (altitude) and angular motion (attitude) are considered. For the design of the controller, the following state variables are defined:

$$x = [z \ \dot{z} \ \phi \ \dot{\phi} \ \theta \ \dot{\theta} \ \psi \ \dot{\psi}]^T = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6 \ x_7 \ x_8]^T \tag{2}$$

The four DOF nonlinear system can be decoupled by assuming a small deviation from hovering condition [15]. Since the simplified system is decoupled, then the altitude and the rotational dynamics of quadrotor can be decomposed into four nonlinear subsystems, which are:

Altitude subsystem:

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= f_1(x) + g_1(x)u_1
\end{aligned} \tag{3}$$

where

$$\begin{aligned}
f_1(x) &= -g \\
g_1(x) &= c_\phi c_\theta \left(\frac{1}{m} \right)
\end{aligned}$$

Roll subsystem:

$$\begin{aligned}
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= f_2(x) + g_2(x)u_2
\end{aligned} \tag{4}$$

where

$$\begin{aligned}
f_2(x) &= \dot{\theta} \dot{\psi} \left(\frac{I_{yy} - I_{zz}}{I_{xx}} \right) - \frac{J_r}{I_{xx}} \dot{\theta} \Omega_d \\
g_2(x) &= \frac{l}{I_{xx}}
\end{aligned}$$

Pitch subsystem:

$$\begin{aligned}\dot{x}_5 &= x_6 \\ \dot{x}_6 &= f_3(x) + g_3(x)u_3\end{aligned}\quad (5)$$

where

$$\begin{aligned}f_3(x) &= \dot{\phi}\psi\left(\frac{I_{zz} - I_{xx}}{I_{yy}}\right) + \frac{J_r}{I_{yy}}\dot{\phi}\Omega_d \\ g_3(x) &= \frac{l}{I_{yy}}\end{aligned}$$

Yaw subsystem:

$$\begin{aligned}\dot{x}_7 &= x_8 \\ \dot{x}_8 &= f_4(x) + g_4(x)u_4\end{aligned}\quad (6)$$

where

$$\begin{aligned}f_4(x) &= \dot{\theta}\phi\left(\frac{I_{xx} - I_{yy}}{I_{zz}}\right) \\ g_4(x) &= \frac{1}{I_{zz}}\end{aligned}$$

Thus the dynamic equations of the quadrotor can be decomposed into four single-input nonlinear subsystems in the form of:

$$x^{(n)} = f(x) + g(x)u, \quad n = 2 \quad (7)$$

where u is the input; $f(x)$ and $g(x)$ are the nonlinear function.

The control objective in this work is to design a suitable control law for the system (7) so that the state vector x of the quadrotor system can track a desired reference trajectory vector x_d . The design of backstepping control for the quadrotor systems is described step-by-step as follows [16]:

Step 1: Define the tracking error:

$$e_1 = x_d - x \quad (8)$$

where x_d is a desired trajectory specified by a reference model. Then the derivative of tracking error can be represented as:

$$\dot{e}_1 = \dot{x}_d - \dot{x} \quad (9)$$

The first Lyapunov function is chosen as:

$$V_1(e_1) = \frac{1}{2}e_1^2 \quad (10)$$

The derivative of V_1 is:

$$\dot{V}_1(e_1) = e_1\dot{e}_1 = e_1(\dot{x}_d - \dot{x}) \quad (11)$$

\dot{x} can be viewed as a virtual control. The desired value of virtual control known as a stabilizing function can be defined as follows:

$$\alpha = \dot{x}_d + k_1e_1 \quad (12)$$

where k_1 is a positive constant. By substituting the virtual control by its desired value, Equation (11) then becomes:

$$\dot{V}_1(e_1) = -k_1e_1^2 \leq 0 \quad (13)$$

Step 2: The deviation of the virtual control from its desired value can be defined as:

$$e_2 = \alpha - \dot{x} = \dot{x}_d + k_1 e_1 - \dot{x} \quad (14)$$

The derivative of e_2 is expressed as:

$$\begin{aligned} \dot{e}_2 &= \dot{\alpha} - \ddot{x} \\ &= k_1 \dot{e}_1 + \ddot{x}_d - f(x) - g(x)u \end{aligned} \quad (15)$$

The second Lyapunov function is chosen as:

$$V_2(e_1, e_2) = \frac{1}{2} e_1^2 + \frac{1}{2} e_2^2 \quad (16)$$

Finding derivative of Equation (16), yields:

$$\begin{aligned} \dot{V}_2(e_1, e_2) &= e_1 \dot{e}_1 + e_2 \dot{e}_2 \\ &= e_1 (\dot{x}_d - \dot{x}) + e_2 (\dot{\alpha} - \ddot{x}) \\ &= e_1 (e_2 - k_1 e_1) + e_2 (k_1 \dot{e}_1 + \ddot{x}_d - f(x) - g(x)u) \\ &= -k_1 e_1^2 + e_2 (e_1 + k_1 \dot{e}_1 + \ddot{x}_d - f(x) - g(x)u) \end{aligned} \quad (17)$$

Step 3: For satisfying $\dot{V}_2(e_1, e_2) \leq 0$, the control input u is selected as:

$$u = \frac{1}{g(x)} (e_1 + k_1 \dot{e}_1 + \ddot{x}_d - f(x) + k_2 e_2) \quad (18)$$

where k_2 is a positive constant. The term $k_2 e_2$ is added to stabilize the tracking error e_1 . Substituting Equation (18) into Equation (17), the following equation can be obtained:

$$\dot{V}_2(e_1, e_2) = -k_1 e_1^2 - k_2 e_2^2 = -E^T K E \leq 0 \quad (19)$$

where $E = [e_1 \ e_2]^T$ and $K = \text{diag}(k_1, k_2)$. Since $\dot{V}_2(e_1, e_2) \leq 0$, $\dot{V}_2(e_1, e_2)$ is negative semi-definite. Therefore, the control law in Equation (18) will stabilize the system.

3. SIMULATION MODEL

As a precursor for developing a model based control design, the simulation environment of the quadrotor mathematical model is developed. The simulation model provides a platform suitable for control design of quadrotor systems to be used for control algorithm development and verification, before working with a real experimental system. For modeling and simulating the quadrotor dynamic model a few steps design procedure need to be done. Firstly, the differential equations in Equation (1) of the quadrotor systems are modeled using Matlab Simulink. The parameters values used in the simulation are taken from Voos [17], as listed in Table 1. Figure 2 depicts the simulink model used for the simulation studies. Initially, the numerical solution of the model is solved using the *ode45* variable-step solver, with a relative tolerance of 0.001 (the default). A variable-step solver can shorten the simulation time significantly because it can dynamically adjust the step size as necessary and thus reduce the number of steps. Even this solver can give a desired level of accuracy, but it is not useful for real-time simulation. Thus, the solver options used in this paper are configured as fixed-step type. To determine the best solver for the simulation model, generally requires experimentation. Following is an efficient way to choose the best fixed-step solver for the simulation model experimentally:

Step 1: Use *ode1* to simulate the simulation model at the specified step size. The simulation results for *ode1* are compared with the simulation for the variable-step solver. If the results are the same for the specified level of accuracy, that is the best fixed-step solver for the simulation model, namely *ode1*. This conclusion can be drawn since *ode1* is the simplest of the fixed-step solvers and hence yields the shortest simulation time for the current step size.

Step 2: If *ode1* does not give satisfactory results, repeat the preceding step with each of the other fixed-step solvers until the one that gives accurate results with the least computational effort is found.

Step 3: If all of the solver do not give accurate results, reduce the simulation step size and repeat from the initial step. Continue in this way until a solver that solves the simulation model accurately with the least computational effort is found.

After performing thorough experiment, the best suited setting for running the simulation model is *ode1* (Euler) fixed-step solver, with a step-size of 0.02 s. The time of 50 s is used to run the simulation. This configuration can handle the simulation model accurately and efficiently with the least computational effort.

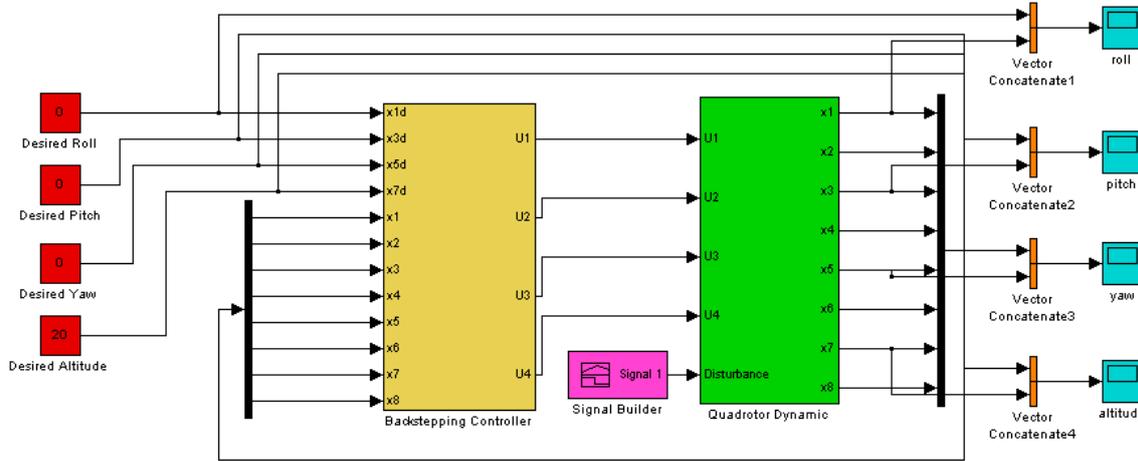


Figure 2. The overall Simulink model of the quadrotor helicopter and the control system

Table 1. Parameters of the quadrotor

Parameter	Description	Value	Units
g	Gravity	9.81	m/s^2
m	Mass	0.5	kg
l	Distance	0.2	m
I_{xx}	Roll inertia	4.85×10^{-3}	$kg \cdot m^2$
I_{yy}	Pitch inertia	4.85×10^{-3}	$kg \cdot m^2$
I_{zz}	Yaw inertia	8.81×10^{-3}	$kg \cdot m^2$
b	Thrust factor	2.92×10^{-6}	
d	Drag factor	1.12×10^{-7}	

4. SIMULATION RESULTS

To explore the effectiveness of the proposed controller, a simulation experiments have been performed on the quadrotor. The control objectives are to reach and maintain quadrotor at a certain desired altitude/attitude, such that the helicopter can hover at a fixed point. The desired altitude/attitude is given by $x_{id} = [z_d, \phi_d, \theta_d, \psi_d] = [20, 0, 0, 0]^T$. The initial states are given by $z = 0$, $\phi = 0.2$, $\theta = 0.2$ and $\psi = 0.2$. Simulation results show the control design is able to stabilize the helicopter in hover mode. Under the proposed controller, it can be observed that the altitude/attitude of the quadrotor can be maintained at the desired altitude/attitude, that is, the hovering flight is stable as shown in Figure 3. Also from this figure, it can be noted that the attitude states converge to zero set-point for a given initial condition rapidly as the system starts, and hence the stabilization of the quadrotor system is achieved.

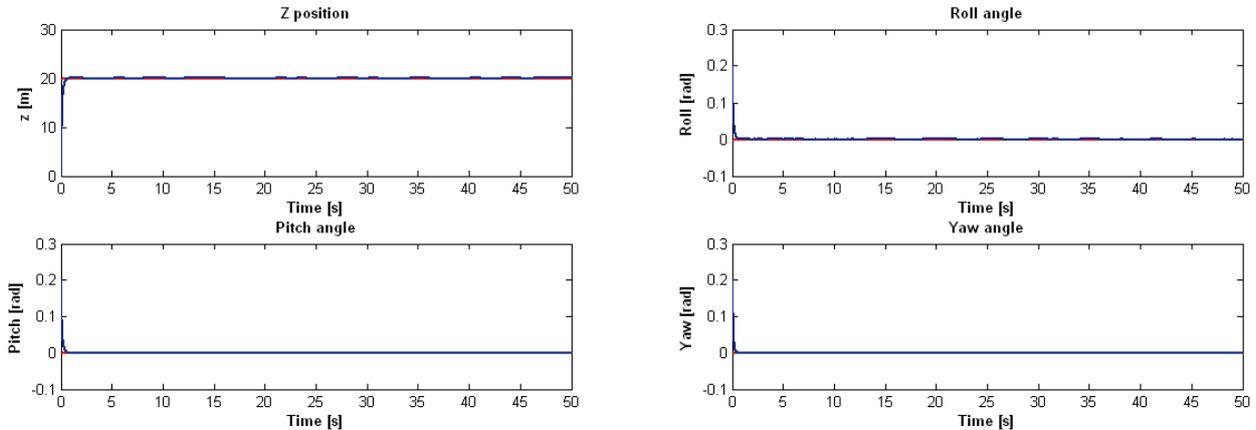


Figure 3. Altitude/attitude of the hovering quadrotor using proposed control

To further highlight the advantage of the proposed control structure the simulated results with different reference trajectory and external disturbance are depicted in Figure 4. The simulation result of altitude tracking response due to periodic rectangular function is shown in Figure 4(a). Figure 4(b) and (c) show the control response of the roll and pitch angle with the target value 0.5 and -0.7 rad, respectively, and roll angle is exerted with an external disturbance ($\phi(30) = 0.2$ rad) at 30 s. Under different setting, the periodic sinusoidal function is used as a reference to the yaw angle and the response is shown in Figure 4(d). As it can be seen, the state trajectory x_i tracks the desired reference trajectory x_{id} quickly. The proposed control can give small tracking error and good tracking performance. Furthermore, the control design can effectively attenuate the external disturbance as can be seen in Figure 4(b).

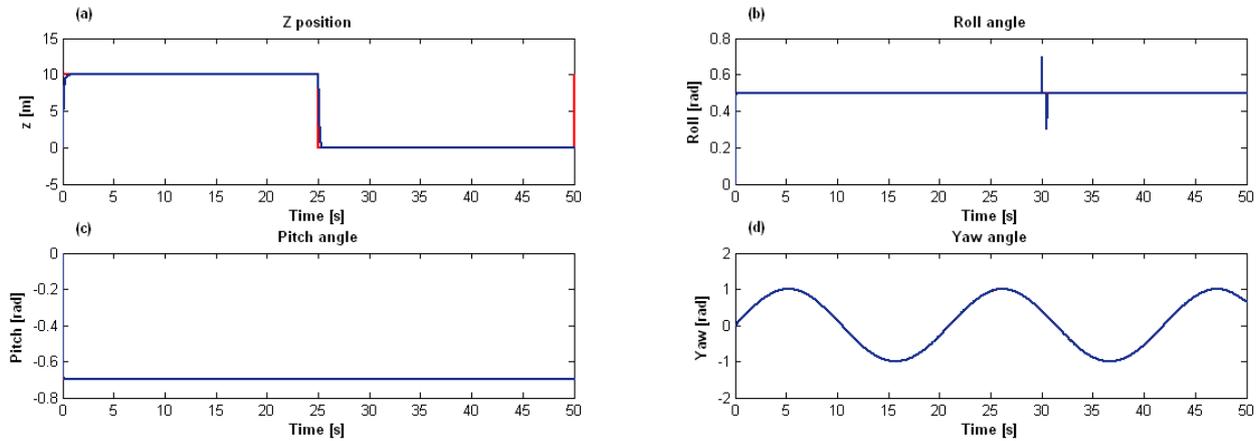


Figure 4. Simulation results using proposed control for (a) altitude tracking response due to periodic rectangular function (b) roll angle with initial value, $\phi(0) = 0$, target value, $\phi_d(t) = 0.5$ and external disturbance, $\phi(30) = 0.2$ (c) pitch angle with initial value, $\theta(0) = 0$ and target value, $\theta_d(t) = -0.7$ (d) yaw tracking response due to periodic sinusoidal function

For the aim of the comparison of the control performance, the proportional-derivative (PD) control is used to control the quadrotor. The parameters of PD controller are heuristically selected as $k_p = 0.2$ and $k_d = 0.7$. Figures 5 and 6 show the simulation results of the PD control to perform stabilization and trajectory tracking problems where it can be seen the settling time with PD controller is rather large and having a small overshoot, but in contrast, the transient response is fast and non-overshooting using the proposed control.

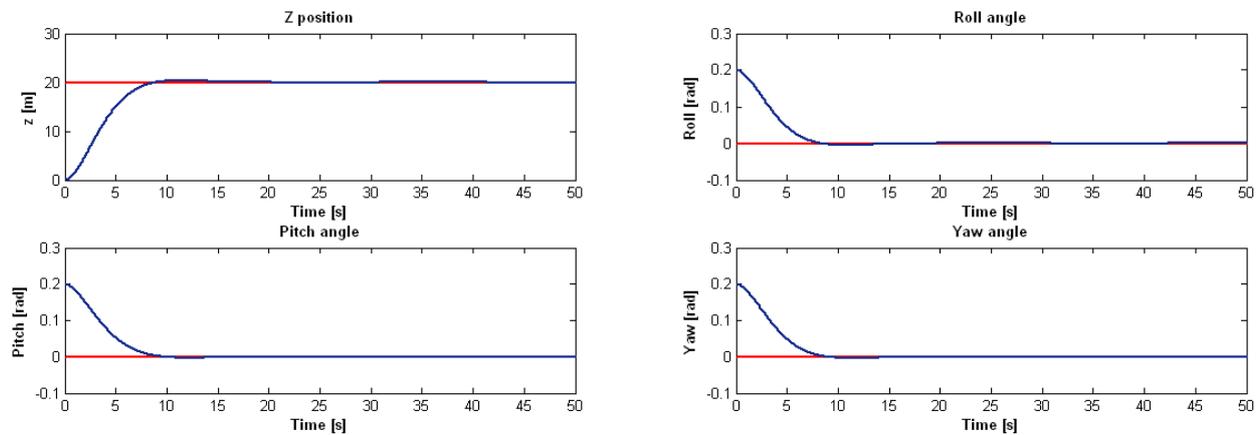


Figure 5. Altitude/attitude of the hovering quadrotor using PD control

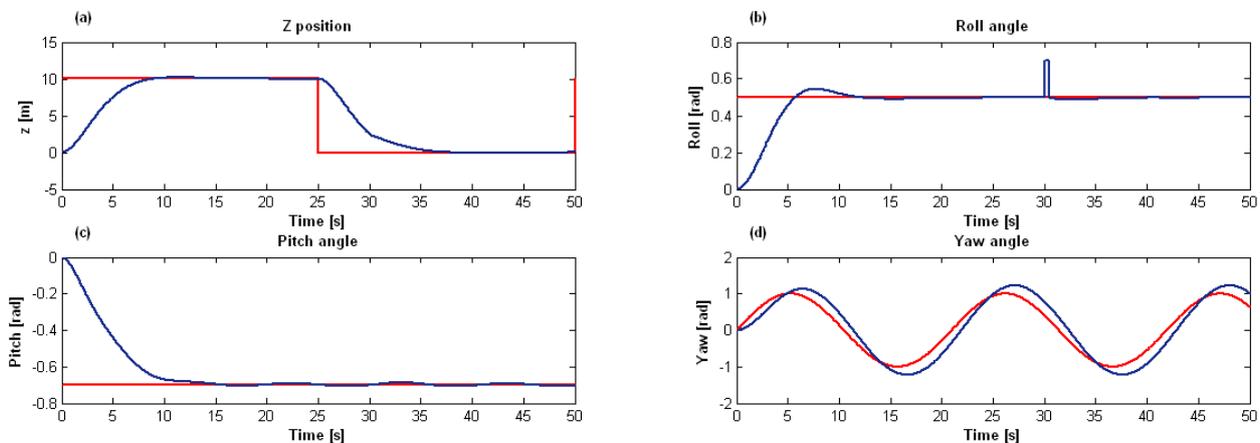


Figure 6. Simulation results using PD control for (a) altitude tracking response due to periodic rectangular function (b) roll angle with initial value, $\phi(0) = 0$, target value, $\phi_d(t) = 0.5$ and external disturbance, $\phi(30) = 0.2$ (c) pitch angle with initial value, $\theta(0) = 0$ and target value, $\theta_d(t) = -0.7$ (d) yaw tracking response due to periodic sinusoidal function

From the simulation results, a satisfactory control performance of the proposed control scheme can be clearly seen under the conditions of external disturbance and various reference trajectories. Since the proposed control structure captures the dynamic response of controlled system, the proposed control will achieve good control performance for the quadrotor system. Obviously, the transient and tracking performance of the proposed method are better than PD control. Therefore, the proposed control scheme is more suitable for autonomous hovering quadrotor.

5. CONCLUSION

In this paper, the application of backstepping controller to control the altitude and attitude of a quadrotor helicopter is successfully demonstrated. First, the mathematical model of the quadrotor is introduced. Then, the proposed controller is developed. The backstepping control design is derived based on Lyapunov function, so that the stability of the system can be guaranteed. Finally, the proposed control scheme is applied to autonomous hovering quadrotor helicopter. Simulation results show that high-precision transient and tracking response can be achieved by using the proposed control system.

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REFERENCES

- [1] T. Hamel, R. Mahony, R. Lozano and J. Ostrowski, Dynamic modelling and configuration and stabilization for an X4-flyer, *International Federation of Automatic Control Proceedings Volumes*, 35(1), 217–222, 2002.
- [2] P. Pounds, R. Mahony, P. Hynes and J. Roberts, Design of a four-rotor aerial robot, *Proc. 2002 Australasian Conference on Robotic and Automation*, Auckland, New Zealand, 2002, pp. 145–150.
- [3] P. McKerrow, Modelling the draganflyer four-rotor helicopter, *IEEE International Conference on Robotics and Automation*, LA, USA, 2004, pp. 3596–3601.
- [4] P. Castillo, R. Lozano and A. Dzul, Stabilization of a mini rotorcraft with four rotors, *IEEE Control Systems Magazine*, 25(6), 45–55, 2005.
- [5] A. L. Salih, M. Moghavvemi, H. A. F. Mohamed and K. S. Gaeid, Modelling and PID controller design for a quadrotor unmanned air vehicle, *IEEE International Conference on Automation Quality and Testing Robotics*, Cluj-Napoca, Romania, 2010, pp. 1–5.
- [6] T. Sangyam, P. Laohapiengsak, W. Chongcharoen and I. Nilkhamhang, Path tracking of UAV using self-tuning PID controller based on fuzzy logic, *Proceedings of SICE Annual Conference*, Taipei, Taiwan, 2010, pp. 1265–1269.
- [7] S. Park, D. H. Won, M. S. Kang, T. J. Kim, H. G. Lee and S. J. Kwon, RIC (robust internal-loop compensator) based flight control of a quad-rotor type UAV, *IEEE/RSJ International Conference on Intelligent Robots and Systems*, Edmonton, Canada, 2005, pp. 3542–3547.
- [8] Z. Tang, L. He and Z. Pei, Control and formation flying for the quadrotor UAV, *International Conference on Photonics, 3D-Imaging, and Visualization International Society for Optics and Photonics*, Guangzhou, China, 2011, pp. 82052–82056.
- [9] M. Tarhan and E. Altug, EKF based attitude estimation and stabilization of a quadrotor UAV using vanishing points in catadioptric images, *Journal of Intelligent & Robotic Systems*, 62(3), 587–607, 2011.
- [10] M. Akhil, M. K. Anand, A. Sreekumar and P. Hithesan, Simulation of the mathematical model of a quad rotor control system using matlab simulink, *Applied Mechanics and Materials*, 110, 2577–2584, 2012.
- [11] M. S. De Queiroz and D. M. Dawson, Nonlinear control of active magnetic bearings: A backstepping approach, *IEEE Transactions on Control Systems Technology*, 4(5), 545–552, 1996.
- [12] Z. J. Wu, X. J. Xie, P. Shi and Y. Xia, Backstepping controller design for a class of stochastic nonlinear systems with markovian switching, *Automatica*, 45(4), 997–1004, 2009.
- [13] M. Krstic, I. Kanellakopoulos and P. Kokotovic, *Nonlinear and adaptive control design*, New York, 1995.
- [14] M. A. M. Basri, K. A. Danapalasingam and A. R. Husain, Design and optimization of backstepping controller for an underactuated autonomous quadrotor unmanned aerial vehicle, *Transactions of Famena*, 38(3), 27–44, 2014.
- [15] H. Huang, G. M. Hoffmann, S. L. Waslander and C. Tomlin, Aerodynamics and control of autonomous quadrotor helicopters in aggressive maneuvering, *IEEE International Conference on Robotics and Automation*, Kobe, Japan, 2009, pp. 3277–3282.
- [16] M. A. M. Basri, A. R. Husain and K. A. Danapalasingam, Backstepping controller with intelligent parameters selection for stabilization of quadrotor helicopter, *Journal of Engineering Science and Technology Review*, 7(2), 66–74, 2014.
- [17] H. Voos, Nonlinear control of a quadrotor micro-UAV using feedback-linearization, *IEEE International Conference on Mechatronics*, Malaga, Spain, 2009, pp. 1–6.